#### Final Report

# COMPUTATIONAL STRATEGY FOR PREDICTING THE DYNAMIC P /2/RESPONSE OF COMPOSITE STRUCTURES DURING IMPACT

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October 1994

#### **PREFACE**

This document contains the proceedings of the Workshop on Computational Methods for Crashworthiness held at NASA Langley Research Center, September 2-3, 1992. The workshop was jointly sponsored by the University of Virginia Center for Computational Structures. Technology and NASA. Workshop attendees came from government agencies, the aerospace and automotive industries, energy laboratories, and universities. The objectives of the workshop were to assess the state-of-technology in the numerical simulation of crash and to provide guidelines for focused future research leading to an enhanced capability for numerical crash simulation.

Certain materials and products are identified in this publication in order to specify adequately the materials and products that were investigated in the research effort. In no case does such identification imply recommendation or endorsement of products by NASA, nor does it imply that the materials and products are the only ones or the best ones available for the purpose. In many cases equivalent materials and products are available and would probably produce equivalent results.

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#### **ABSTRACT**

The objective of the proposed study is to develop an effective computational strategy for predicting the nonlinear vibrations, and dynamic response of large composite airframe structural components during impact. The two key elements of the proposed strategy are: a) use to a novel hierarchical modeling strategy, and b) evaluation of the sensitivity of the nonlinear vibration and dynamic response to the various parameters characterizing the model. The first level in the strategy uses thin-walled beam finite element model which incorporates the effects of warping and distortion of the cross section. The characteristic response quantities, which identify the critical region of the structure are calculated. In the second level, two-dimensional shell and plate finite element models are used to determine the detailed response and, possibly, failure characteristics in the critical region. Based on the sensitivity information, the effectiveness of using hybrid numerical/experimental simulation of the response will be explored.

The structures considered in this study are of the generic type (thin-walled beam, composite panels, tubes and cylinders) for which experimental data is either available or will become available in the course of the study.

The Principal Investigator is Dr. Ahmed K. Noor, Ferman W. Perry Professor of Aerospace Structures and Applied Mechanics, and the NASA Technical Officer is Mr. Huey D. Carden, Aerospace Engineer, Structural Mechanics Branch, Structures Division, NASA Langley Research Center.

#### SUMMARY OF RESEARCH WORK

During the period of this grant (Nov. 8, 1990-Nov. 7, 1993), the research was focused on developing an efficient reduced basis technique and a computational procedure for predicting the nonlinear vibrational response of composite frames. The computational procedure can be conveniently divided into two distinct phases. The first phase involves the generation of basis vectors using the Linstedt-Poincare perturbation technique. The second phase consists of computing the amplitudes of the basis vectors, and the nonlinear frequency of vibration via a direct variational procedure.

The analytical formulation is based on a form of this moderate rotation, geometrically nonlinear thin-walled beam theory and Sanders-Budiansky type theory of shells with the effects of transverse shear deformation and anisotropic material behavior included. The frame is discretized by using truncated Fourier series for thin-walled beams, and mixed, two-dimensional shell finite element model with the fundamental unknowns consisting of both the nodal displacements and the stress-resultant parameters.

The linear vibration frequencies are obtained by solving a linear eigenvalue problem. For each pair of eigenvalue-eigenvector, Linstedt-Poincare method is used to generate perturbation vectors. This is accomplished by expanding each of the vectors of displacement amplitudes for thin-walled beams, and nodal displacements and stress-resultant parameters in two-dimensional finite element model in a perturbation series; and expanding the time-dependent vector coefficients of the series into a Fourier cosine series in a time parameter. The perturbation vectors are then selected as basis vectors and the Rayleigh-Ritz technique is applied, in conjunction with the harmonic balance method, to generate a set of reduced nonlinear algebraic equations, which approximate the original semi-discrete finite element equations of the structure. The unknowns in these equations are the amplitudes of the basis vectors and the nonlinear frequency of vibration.

The accuracy of the predictions of the reduced basis technique has been demonstrated by a number of numerical examples of thin-walled beams, multilayered composite plates and composite frames. The results are reported in Refs. 1-5. Also, a workshop was organized by the principal investigator on "Computational Methods for Crashworthiness." The proceedings of the workshop have been published as a NASA CP (Ref. 6).

#### PAPERS PUBLISHED UNDER NASA GRANT NAG-1-1197 (1990-1994)

- 1. Noor, A. K., Andersen, C. M. and Peters, J. M., "Reduced Basis Technique for Nonlinear Vibration Analysis of Composite Panels," Computer Methods in Applied Mechanics and Engineering, Vol. 103, 1993, pp. 175-186.
- 2. Noor, A. K., Hadian, M. J. and Andersen, C. M., "Hybrid Analytical Technique for the Nonlinear Vibration Analysis of Thin-Walled Beams," Journal of Engineering Mechanics, ASCE, Vol. 119, No. 4, April 1993, pp. 786-800.

- X 3. Noor, A. K., Hadian, M. J. and Andersen, C. M., "Hybrid Analytical Technique for Evaluating the Sensitivity of the Nonlinear Vibrational Response of Beams," Computers and Structures, Vol. 49, No. 4, Nov. 1993, pp. 569-577.
  - 4. Noor, A. K., Hadian, M. J. and Peters, J. M., "Reduced Basis Technique for Evaluating the Sensitivity of the Nonlinear Vibrational Response of Composite Plates," Computers and Structures, Vol. 52, No. 6, 1994, pp. 1097-1106.
- X5. Noor, A. K. and Peters, J. M., "Nonlinear Vibrations of Thin-Walled Composite Frames," Shock and Vibration Journal (to appear).
  - 6. Noor, A. K. and Carden, H. D. (compilers), Computational Methods for Crashworthiness, Proceedings of the Workshop, Sept. 2-3, 1992, NASA Langley Research Center, Hampton, VA, NASA CP-3223, Aug. 1993.

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## Reduced basis technique for nonlinear vibration analysis of composite panels

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Received 15 October 1992 Revised manuscript received 25 November 1992

A reduced basis technique and a computational procedure are presented for the nonlinear free vibrations of composite panels. The computational procedure can be conveniently divided into two distinct steps. The first step involves the generation of various-order perturbation vectors using Linstedt-Poincaré perturbation technique. The second step consists of using the perturbation vectors as basis vectors, computing the amplitudes of these vectors, and the nonlinear frequency of vibration, via a direct variational procedure. The analytical formulation is based on a form of the geometrically nonlinear shallow shell theory with the effects of transverse shear deformation, rotatory inertia and anisotropic material behavior included. The panel is discretized by using mixed finite element models with the fundamental unknowns consisting of both the nodal displacements and the stress-resultant parameters of the panel. The potential of the proposed technique is discussed and its effectiveness is demonstrated by means of numerical examples.

#### Hybrid Analytical Technique for Nonlinear Vibration Analysis of Thin-Walled Beams

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ABSTRACT: A two-step hybrid analytical technique is presented for the nonlinear vibration analysis of thin-walled beams. The first step involves the generation of various-order perturbation functions using the Linstedt-Poincaré perturbation technique. The second step consists of using the perturbation functions as coordinate (or approximation) functions and then computing both the amplitudes of these functions and the nonlinear frequency of vibration via a direct variational procedure. The analytical formulation is based on a form of the geometrically nonlinear beam theory with the effects of in-plane inertia, rotatory inertia, and transverse shear deformation included. The effectiveness of the proposed technique is demonstrated by means of a numerical example of thin-walled beam with a doubly symmetric I-section. The solutions obtained using a single-spatial mode were compared with those obtained using multiple-spatial modes. The standard of comparison was taken to be the frequencies obtained by the direct integration/fast Fourier transform (FFT) technique. The nonlinear frequencies obtained by the hybrid technique were shown to converge to the corresponding ones obtained by the direct integration/fast Fourier transform (FFT) technique well beyond the range of applicability of the perturbation technique. The frequencies and total strain energy of the beam were overestimated by using a single-spatial mode.

## HYBRID ANALYTICAL TECHNIQUE FOR EVALUATING THE SENSITIVITY OF THE NONLINEAR VIBRATIONAL RESPONSE OF BEAMS

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Abstract—A three-step hybrid analytical technique is presented for evaluating the nonlinear vibrational response as well as the first-order sensitivity coefficients of thin-walled beams (derivatives of the nonlinear frequency with respect to material and geometric parameters of the beam). The first step involves the generation of various-order perturbation functions, and their derivatives with respect to the material and geometric parameters of the beam, using the Linstedt-Poincaré perturbation technique. The second step consists of using the perturbation functions as coordinate (or approximation) functions and then computing the amplitudes of these functions and the nonlinear frequency via a direct variational procedure. The third step consists of using the perturbation functions, and their derivatives, as coordinate functions and computing the sensitivity coefficients of the nonlinear frequency via a second application of the direct variational procedure. The analytical formulation is based on a form of the geometrically nonlinear beam theory with the effects of in-plane inertia, rotatory inertia, transverse shear deformation, and cross-sectional warping included. The effectiveness of the proposed technique is demonstrated by means of a numerical example of thin-walled beam with a doubly symmetric I-section.



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#### REDUCED BASIS TECHNIQUE FOR EVALUATING THE SENSITIVITY OF THE NONLINEAR VIBRATIONAL RESPONSE OF COMPOSITE PLATES

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(Received 28 November 1993)

Abstract—A reduced basis technique and a computational procedure are presented for generating the nonlinear vibrational response, and evaluating the first-order sensitivity coefficients of composite plates (derivatives of the nonlinear frequency with respect to material and geometric parameters of the plate). The analytical formulation is based on a form of the geometrically nonlinear shallow shell theory with the effects of transverse shear deformation, rotatory inertia and anisotropic material behavior included. The plate is discretized by using mixed finite element models with the fundamental unknowns consisting of both the nodal displacements and the stress-resultant parameters of the plate. The computational procedure can be conveniently divided into three distinct steps. The first step involves the generation of various-order perturbation vectors, and their derivatives with respect to the material and lamination parameters of the plate, using Linstedt-Poincaré perturbation technique. The second step consists of using the perturbation vectors as basis vectors, computing the amplitudes of these vectors and the nonlinear frequency of vibration, via a direct variational procedure. The third step consists of using the perturbation vectors, and their derivatives, as basis vectors and computing the sensitivity coefficients of the nonlinear frequency via a second application of the direct variational procedure. The effectiveness of the proposed technique is demonstrated by means of numerical examples of composite plates.

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### Nonlinear Vibrations of Thin-Walled Composite Frames

A reduced basis technique and a computational procedure are presented for generating the nonlinear vibrational response, and evaluating the first-order sensitivity coefficients of thin-walled composite frames. The sensitivity coefficients are the derivatives of the nonlinear frequency with respect to the material and lamination parameters of the frame. A mixed formulation is used with the fundamental unknowns consisting of both the generalized displacements and stress resultants in the frame. The flanges and webs of the frames are modeled by using geometrically nonlinear two-dimensional shell and plate finite elements. The computational procedure can be conveniently divided into three distinct steps. The first step involves the generation of various-order perturbation vectors, and their derivatives with respect to the material and lamination parameters of the frame, using the Linstedt-Poincaré perturbation technique. The second step consists of using the perturbation vectors as basis vectors, computing the amplitudes of these vectors and the nonlinear frequency of vibration, via a direct variational procedure. The third step consists of using the perturbation vectors, and their derivatives, as basis vectors and computing the sensitivity coefficients of the nonlinear frequency via a second application of the direct variational procedure. Numerical results are presented for semicircular thin-walled frames with I and J sections, showing the convergence of the nonlinear frequency and the sensitivity coefficients obtained by both the reduced-basis and perturbation techniques ( 1994 John Wiley & Sons, Inc.

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